

The Use of Box-Cox Transformation Technique in Economic and Statistical Analyses

Mohammad Zakir Hossain

Department of Operations Management and Business Statistics
 College of Commerce and Economics, Sultan Qaboos University, Oman

Abstract

This paper provides an analytical review of the significant role of the Box-Cox transformation technique in a variety of statistical fields such as estimation, testing, inference and model selection. Although the Box-Cox transformation technique has been extensively studied during the past few decades, any analytical bibliography, particularly in the context of model selection, is yet to emerge. An attempt is made in this paper to bridge this gap

Keywords: Box-Cox transformation; functional forms; model selection; estimation; hypothesis test

INTRODUCTION

The statistical techniques that are often used in linear regression models are founded upon three basic assumptions namely, linearity, normality and homoscedasticity. The first assumption – linearity – helps interpretation of the data and identification of the properties of the parameters of the model in question. Normality helps to apply the standard statistical testing procedures comfortably in a number of statistical analyses. Finally, the assumption of homoscedasticity helps in several ways such as:

- conducting tests of hypothesis and construct confidence intervals through the use of the formula of the variances of the coefficients,
- making unbiased and efficient OLS estimates, both in small and large samples,
- predicting the variable of interest for certain given values of the explanatory variables,
- Simplifying the standard estimation techniques.

The violation of the assumptions discussed above creates a number of unwelcome problems in most economic and statistical analyses. See, for example, Hoyle (1973), Koutsoyiannis (1977), Sakia (1992), and Hossain (1998). In order to overcome these problems, Graybill (1976) provided four important suggestions, of which *transformations of variables* has probably attracted most attention. See, for example, Theoeni (1967), Hoyle (1973), and Sakia (1992). In fact, the purpose of a transformation is to improve the quality of approximation of the proposed models. In this regard Tukey (1957) argues that “transformation of variables may lead to a more nearly linear model, may stabilize the error variance, and/or may lead to a model for which a symmetrically, perhaps normally distributed error term is acceptable”. A number of transformation techniques have been proposed in the literature. For example, linear transformation, log transformation, reciprocal transformation, square root transformation,

cube root transformation, arc sign transformation, arc tanh transformation, angular transformation, log-log transformation, coordinate transformation, probit transformation, logit transformation, monotone transformation, Johnson’s transformation, power transformation, and so on (Hoyle, 1973). Statisticians, economists and econometricians use different transformations for different purposes. Since the seminal paper by Box and Cox (1964), a number of Box-Cox type of power transformations have been generated, both in theoretical work and in practical applications. Of them, Manly (1976) proposes the following exponential data transformation:

$$y(\lambda) = \frac{e^{\lambda y} - 1}{\lambda}, \quad \text{if } \lambda \neq 0, \\ = y, \quad \text{if } \lambda = 0, \quad (1)$$

where negative value of y could be allowed. The transformation was reported to be useful in order to transform unimodal skewed distribution into normal distribution, but has not been useful for bimodal or U-shaped distribution. John and Draper (1980), on the other hand, proposed the following modification of the above power transformation which they called “Modulus Transformation”.

$$y(\lambda) = \text{sign}(y) \frac{(|y| + 1)^\lambda - 1}{\lambda}, \quad \text{if } \lambda \neq 0, \\ = \text{sign}(y) \log(|y| + 1), \quad \text{if } \lambda = 0,$$

Where

$$\text{Sign}(y) = 1, \quad \text{if } y \geq 0, \\ = -1, \quad \text{if } y < 0. \quad (2)$$

Here negative value of y could be allowed. Modular transformation works best at those distributions that are somewhat symmetric. A power transformation on a symmetric distribution is likely to introduce some

degree of skewness. Bickel and Doksum (1981) gave the following slight modification in their examination of the asymptotic performance of the parameters in the Box-Cox transformations model:

$$y(\lambda) = \frac{|y|^\lambda \text{Sign}(y) - 1}{\lambda}, \quad \text{if } \lambda > 0,$$

Where. $\text{Sign}(y) = 1, \quad \text{if } y \geq 0,$
 $\quad \quad \quad = -1, \quad \text{if } y < 0$ (3)

Recently, Yeo and Johnson (2000) have proposed a new family of distributions that can be used without restrictions on y and that have many of the good properties of the Box-Cox power family. These transformations are defined as:

$$y(\lambda) = \frac{(y+1)^\lambda - 1}{\lambda} \quad \text{if } \lambda \neq 0, \quad y \geq 0;$$

$$= \log(y+1), \quad \text{if } \lambda = 0, \quad y \geq 0;$$

$$= \frac{(1-y)^{2-\lambda} - 1}{\lambda - 2} \quad \text{if } \lambda \neq 2, \quad y < 0;$$

$$= -\log(1-y), \quad \text{if } \lambda = 0, \quad y < 0. \quad (4)$$

For estimating the transformation parameter, they found the value of λ that minimizes the Kullback-Leibler distance between the normal distribution and the transformed distribution.

The power transformations discussed above have been found to be very useful in the empirical determination of functional relationships in many economic and statistical applications. Of these transformations, the Box-Cox transformation has still been the most commonly used power transformation in statistics, economics and econometrics, see for example, Sakia (1992), Davidson and MacKinnon (1993), Hossain and Bhatti (2003) and Hossain and King (2008). The Box-Cox power transformation can be defined as:

$$y(\lambda) = \frac{y^\lambda - 1}{\lambda}, \quad \text{if } \lambda \neq 0,$$

$$= \ln(y), \quad \text{if } \lambda = 0,$$

$$= y - 1, \quad \text{if } \lambda = 1, \quad (5)$$

where the value of y must be greater than zero and the transformation parameter λ can take any real values.

Sakia (1992) argued that among these various power transformations, the Box-Cox transformation technique can be more successfully applied in the functional relationship between variables of interest for economic and statistical models. Gujarati (1992), on the other hand, indicates that by using this transformation technique, one can easily apply specification error tests to determine the actual functional form of a regression model. In general, it is assumed that for each λ , $y(\lambda)$ is a monotonic function of y over the admissible range (Box and

Cox, 1964). Spitzer (1978) commented that the Box-Cox transformation can be regarded as a valuable tool to counter the problem of subjectivity in the model building process. He also indicated that for forecasting purposes, the use of the Box-Cox transformation has emerged as a very promising procedure. Because of such tremendous importance of Box-Cox transformation, we have been motivated to discuss its various applications from the analytical point of view. The rest of this paper is organized as follows. Sections 2 and 3 briefly outline the role of Box-Cox transformation technique in hypothesis testing as well as estimation and model selection. Estimation of the Box-Cox parameter is discussed in section 4. Section 5 concludes the paper.

The Role of Box-Cox Transformation Technique in Hypothesis Testing and Estimation

There has been much work on using the Box-Cox transformation in hypothesis testing and estimation. Andrews (1971) was, perhaps, the pioneer to propose a hypothesis test for the consistency of the data by using the Box-Cox type power transformation within a given family of distributions. As claimed by Andrews, the test is exact, simple, and the power of the test may be useful to measure the quality of inferences. This test was formulated by ignoring the Jacobean of the transformation. Later, this omission was investigated by Atkinson (1973) and he proposed another test by incorporating the Jacobean term. The latter test was found to be more powerful than the Andrews' test. A robust method for testing transformations to achieve normality has been proposed by Carroll (1980) and found to be competitive to these tests. Aneuryn-Evans and Deaton (1980) indicated that if a model is linear with normal errors then there is some probability that negative values of the dependent variable may occur which is not convenient for the log-linear model and hence must be incorrect. This happens in many situations. In order to overcome this problem, they proposed that the error terms be assumed to have a suitable truncated normal distribution and then be applied to the important results (Cox 1961, 1962) on testing nonnested models. However, Aneuryn-Evans and Deaton tests need to estimate only two competing models and are capable of rejecting both when neither is true. One important demerit of their approach is that it involves two test statistics which are rather difficult to calculate. To overcome this, Godfrey and Wickens (1981) proposed a more general test procedure which allows both models, either models or neither model to be rejected. Therefore, as claimed by the authors, their proposed procedure is a more useful addition to existing procedures. Lawrance (1987a) modified Atkinson's (1973) test statistic to test whether the estimate of the Box-Cox transformation parameter conforms to a hypothesized value or not and reported a simulation experiment that compared his proposed test with

Atkinson's test. The study indicated that the modified statistic has improved standard normal behavior relative to Atkinson's test. Lawrance (1987b) has also given a more reasonable expression for the estimated variance of the parameter λ that leads to more efficient hypothesis tests on λ than Atkinson's test procedure (see for example, Sakia, (1992)). Another simulation experiment conducted by Atkinson and Lawrance (1989) showed that, in general, both of their tests are very similar to each other although for small samples the Lawrance test was found to be superior to Atkinson's test. However, Wang (1987) has suggested a more recent improvement of both Atkinson's test and Lawrance's test. He claims that his suggested method gives more accurate approximations to the standard normal.

Peltier et al. (1998) used the Box-Cox transformation algorithm for hypothesis testing in animal science experiments. They found the Box-Cox transformation technique for transforming data is quite useful to satisfy the assumptions of ANOVA. Their study examined whether melatonin implantation would affect progesterone secretion in cycling pony mares and found overall treatment variances were greater in the melatonin-treated group.

Foster et al. (2001) proposed a simple semiparametric estimation method for the Box-Cox transformation model with no specific parametric assumption on the distribution of error term. The resulting estimators are found to be very consistent and asymptotically normal. Their covariance matrix can be estimated through a novel resampling method without involving nonparametric function estimates of the underlying unknown density function for the error term. The new proposal is illustrated with a well-known data set in the literature of transformation, and its efficiency and robustness are closely examined through numerical studies. Elia and Piccolo (2004) studied the interaction between the estimation of the fractional differencing parameter d of ARFIMA models and the common practice of instantaneous transformation of the observed time series. The authors first discussed the effect of a nonlinear transformation of the data on the identification of the process and on the estimate of d . Then, they proposed a joint estimation of the Box-Cox parameter and d by means of a modified normalized version of the Whittle likelihood. They obtained the variance and covariance matrix of the parameters' estimates. Finally, a Monte Carlo study was performed to check the small sample behavior of the proposed estimators and found promising results. Unver et al. (2004) investigated the effects of Box-Cox transformation on estimations of the genetic parameters for egg production traits that do not hold to the assumptions of parametric statistical analysis. Egg production of 1980 animals from 43 sires and 8 dams per sire were used considering five different ages of eggs namely, 22-30, 31-34, 22-34, 31-40 and

22-40 weeks of age. The egg production traits showed non-normal distributions in the form of negative skewness and positive kurtosis. The greatest departure from normality was observed in the peak period (31-34 weeks of age). After applying Box-Cox transformation, the distribution of the transformed data became very closer to normality. Applying Box-Cox transformation to partial egg production data resulted in an increase in heritability and slightly changed genetic and phenotypic correlations. Nevertheless, the Box-Cox transformation is preferred for estimating genetic parameters from the data, when the assumptions are not satisfied.

Ning and Finch (2004) studied the alternative distribution of the likelihood ratio test in which the null hypothesis postulates that the data are from a normal distribution after a restricted Box-Cox transformation and the alternative hypothesis postulates that they are from a mixture of two normals after a restricted Box-Cox transformation. The simulation results demonstrated that power of the test decreases as the mixing proportion differs from a certain tolerance level. Zhang and King (2004) present a Markov chain Monte Carlo (MCMC) algorithm to estimate parameters and latent stochastic processes in the asymmetric stochastic volatility (SV) model, in which the Box-Cox transformation of the squared volatility follows an autoregressive Gaussian distribution and the marginal density of asset returns has heavy-tails. To test for the significance of the Box-Cox transformation parameter, we present the likelihood ratio statistic, in which likelihood functions can be approximated using a particle filter and a Monte Carlo kernel likelihood. When applying the heavy-tailed asymmetric Box-Cox SV model and the proposed sampling algorithm to continuously compounded daily returns of the Australian stock index, we find significant empirical evidence supporting the Box-Cox transformation of the squared volatility against the alternative model involving a logarithmic transformation.

The Box-Cox quantile regression model using the two stage method provided a flexible and numerically attractive extension of linear quantile regression techniques. However, the objective function in stage two of the method may not exist in most situations. To resolve this problem, Fitzenberger et al. (2005) suggested a simple modification of the estimator through Box-Cox transformation which is easy to implement. The modified estimator has been found to be consistent and they derived its asymptotic distribution. A simulation study confirmed that the modified estimator works well in situations, where the original estimator is not well defined.

Terasaka and Hosoya (2007) proposed a new version of the Box-Cox transformation and investigated how it works in terms of asymptotic performance and application, focusing in particular on inference on

stationary multivariate ARMA models. They used a computational estimation procedure which extended the three-step Hannan and Rissanen method. They accommodated the Box-Cox transformation and, for the purpose of parameter testing, they proposed a Monte-Carlo Wald test. They used bivariate series of the Tokyo stock-price index (Topix) and the call rate for their analysis and found very promising results.

Shin (2008) proposed a semiparametric estimation procedure for the Box-Cox transformation model and found that the proposed estimator minimizes a second order U-process and does not require any smoothing parameter that sometimes induces unstable inference result. The proposed estimator can also be applied to random censoring and converges to an asymptotic normal distribution. The Monte Carlo experiments demonstrate adequate finite sample performance.

The Role of Box-Cox Transformation Technique in Model Selection

Unlike physical, chemical or biological scientists, statisticians, economists and econometricians are often forced to work with non-experimental data, which adds various complications to statistical, economic and econometric model building process in practice. In such a case they usually expect that economic theory will help them to find causal links and formulate desired models. But unfortunately, existing economic theory often fails to suggest an adequate functional form of such relationships. Because of this weakness in economic theory, they often use their own subjective judgment in deciding on the functional forms of models. In doing so, they have proposed different alternative forms to reflect the relationship between dependent and independent variables. But the question arises as to how one model should be selected as the correct form from a number of alternative possible models. In our view, the issue of determining the correct functional form within the framework of standard model selection scheme would possibly be able to answer this question properly. Let us now turn to the critic roles of the Box-Cox transformation technique in determining the functional relationships between some variables of interest in statistics, economics and econometrics disciplines. Sargan (1964) was, perhaps, the pioneer to propose a criterion for choosing between two linear regression models based upon the ratio of the maximized likelihoods which is, in fact, the likelihood ratio test and can be defined as

$$S = \left(\frac{\hat{\sigma}_1}{g \hat{\sigma}_2} \right)^r \quad (6)$$

where $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are two estimated standard deviations of random terms included in the linear and log-linear models respectively, g is the geometric mean of the dependent variable y of the linear regression model. The main advantage of Sargan's criterion is that it is very simple to compute from the

ordinary least square (OLS) estimates of the two competing models which lead to a clear-cut decision between them. However, the criterion becomes less useful if neither model is correct. This is one of the main drawbacks of this approach.

Zarembka (1968) considered the functional forms in the demand for money with the same λ value for dependent and independent variables. Hechman and Polschek (1974) examined the functional relationship between earning, schooling and experience with the same λ for all variables. Spitzer (1976) studied the functional relationship between the demand and the liquidity trap with a generalized Box-Cox parameter. Khan and Ross (1977), on the other hand, determined the aggregate functional form of the import demand equation with same λ . Newman (1977) considered the functional relationship between the incidence of malaria and the mortality rate and concluded that the functional form obtained by using the Box-Cox procedure was superior to earlier forms. Mills (1978) investigated the functional form of the UK demand for money. Chang (1980) also investigated the functional relationships of the US demand for meat with considering different λ , for dependent and independent variables in the model. Boylan and Muirheartaigh (1981) presented a critical analysis on the paper "The functional form of the UK demand for money" by Mills (1978). Miner (1982) has done a detailed investigation on Soybean yield functions by using the BCT technique in his Ph.D. dissertation. The author found that the Box-Cox transformation is able to provide approximately normally distributed error term which is prerequisite for conducting hypothesis tests and constructing confidence intervals.

Lin and Huang (1983) illustrated a general approach via application of the Box-Cox transformation technique to estimate national yield trends for corn, wheat and soybean during the period 1960 through 1979. The generalized Box-Cox transformation has also been employed to model price changes by Milton et al. (1984). In this context, some other notable examples where the Box-Cox transformation has been used in determining the functional forms of models include While (1972), Kau and Lee (1976), Boylan et al. (1980), Hwang (1981), Spitzer (1982a, 1982b), Bessler et al. (1984), Davison et al. (1989), He and Shen (1997) and Chen and Pounds (1998). Hossain and King (2003) developed a new forms of AIC for model selection by using the Box-Cox transformation technique where the Box-Cox transformation parameter λ is restricted to be in the range $[0,1]$. A Monte Carlo simulation study to compare their proposed methods with the existing AIC and BIC methods was also presented in their paper. The results supported the proposition that the use of the knowledge of restricted parameters can

lead to significant improvements in information criteria based model selection procedures.

In order to predict a reasonable future lifetime based on a sample of past lifetimes, Yang (2004) employed the Box-Cox transformation method which provides a simple and unified procedure and outperforms the corresponding solution in terms of coverage probability and average length of prediction intervals. The author used Kullback-Leibler information for second-order asymptotic expansion to justify the Box-Cox procedure. Extensive Monte Carlo simulations are also performed to evaluate the small sample behavior of the procedure and found interesting results. Kosei (2006) presented an information criterion based model selection procedure for forecasting purpose that allows both the unit root detection and the Box-Cox transformation simultaneously. Simulation results suggest that the BIC outperforms the bias-corrected AIC and that the augmented Dickey-Fuller test performs worse in the case of incorrect data transformation. Gon and Meddahi (2008) studied the accuracy of a new class of transformations for realized volatility based on the Box-Cox transformation. This transformation is indexed by a parameter λ , and contains as special cases the log (when $\lambda = 0$) and the raw (when $\lambda = 1$) versions of realized volatility. Based on the theory of Edgeworth expansions, they studied the accuracy of the Box-Cox transformation across different values of λ and derived an optimal value of λ that approximately eliminated skewness. They showed that the corresponding Box-Cox transformed statistic outperformed the other choices of λ , including $\lambda = 0$ (the log transformation). The authors provided extensive Monte Carlo simulation results to compare the finite sample properties of different Box-Cox transformations and found the best choice of transformation by controlling the coverage probability of 95% level confidence intervals for integrated volatility. Hossain and King (2008) proposed two model selection procedures in the context of Box-Cox transformations and their application to the linear regression model. Monte Carlo simulation results indicated that their proposed procedures clearly dominate existing procedures in terms of having higher probabilities of correctly selecting the true model.

Estimation of the Box-Cox Parameter

The main objective in the analysis of Box-Cox transformation technique is to estimate and make inference on the transformation parameter λ , and for this, Box and Cox (1964) considered two approaches. The first approach is to use the maximum likelihood (ML) method which is commonly used as it is conceptually easy and the profile likelihood function is easy to compute in this case. It is also easy to

obtain an approximate confidence interval for λ because of the asymptotic property of maximum likelihood estimator. The second approach outlined in Box and Cox (1964) is to use Bayesian method where we need to first ensure that the model is fully identifiable. A transformation parameter can also be estimated on the basis of enforcing a particular assumption of the linear model. For example, if we want to ensure the additivity or linearity in the linear model, we can select a transformation parameter that will minimize the F-value for the degree of freedom for non-additivity. This idea was first expressed in Tukey (1949). Draper and Cox (1969), on the other hand, provided an effective approximation for the precision of ML estimation of λ . However, this approximation method was revised by Hinkley (1975). In contrast to above, Cressie (1978) proposed a simple graphical procedure for estimating λ . For the same purpose, a number of robust procedures have also been prescribed by several investigators such as, Carroll (1980), Bickel and Doksum (1981) and Carroll and Ruppert (1987). Hinkley (1985) on the other hand, advocated for a more analytical procedure for estimating λ . Han (1987) suggested a non-parametric estimator of λ by employing the Kendall's rank correlation approach. An effective computer program called constrained optimization technique for the estimation of λ has extensively been used by Hossain (1998) throughout his Ph. D. dissertation.

Much research on the estimation of λ has been done after Bickel and Doksum (1981), either on a philosophical or on a technical level. Although there does not appear to be any definite result, most research agrees that while there is an effect on not knowing the true value of λ , its cost may not be large enough to discredit the conventional application based on situations.

CONCLUSION

The main aim of the Box-Cox transformations is to ensure that the usual assumptions for linear model to be satisfied. Unfortunately, we sometimes find some typical data that are not being power-transformed to normal. Draper and Cox (1969) addressed this problem and concluded that even in cases where no power-transformation can bring the distribution to exactly normal, the usual estimates of λ will lead to a distribution that satisfies certain restrictions on the first four moments and thus will be usually symmetric. Despite these, there are still a few difficulties with the procedure. For example, the error term cannot be strictly normal since a power transformation does not in general permit negative real values. In such cases, one possible solution is to assume approximate normality for the successful implementation of the procedure. However, the

introduction of the Box-Cox transformation technique has resulted in a much wider range of applications compared with other power transformation techniques in the literature. It has been successful to initiate a huge volume of research in various fields of statistics, economics and econometrics such as estimation, hypothesis testing, inferences, predictions, determination of functional forms and model selection. A brief discussion on these issues has been provided in this paper. In particular, an effort has been made to examine how this transformation technique can be used in the model selection context. An empirical example has also been presented in this paper.

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